Aberdeen Grammar School **Mathematics Department**

CfE Advanced Higher Mathematics Learning Intentions and Success Criteria

BLOCK 1				BLOCK 2				BLOCK 3			
Pages	**Topic**	Specimen Paper	Exemplar Paper	Pages	**Topic**	Specimen Paper	Exemplar Paper	Pages	**Topic**	Specimen Paper	Exemplar Paper
1	Partial Fractions	15(a)		9-11	Integration	3,5,11	15	19	Gaussian Elimination		3
2-4	Differentiation	1,10	2,6	12	Volumes of Revolution		13	20-21	Matrices	6	7,11
5	Differentiation Related Rates	7		13-14	Sequences and Series	9		22	Euclidean Algorithm	4	
5	Differentiation Rectilinear Motion		7	15	McLaurin Series	8	8	22	Methods of Proof	12	9
6	Binomial Theorem	2	1	16-17	Properties of Functions	13	10,14	23-24	Vectors	14	16
7-8	Complex Numbers	17	5	18	Summation Proof by Indication	16	12	25-26	Differential Equations	15(b)	17,18

CfE Advanced Higher Mathematics Formulae List

Standard derivatives

$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals

$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)+c$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Arithmetic series $S_n = \frac{1}{2}n\left[2a+(n-1)d\right]$

Geometric series $S_n = \frac{a(1-r^n)}{1-r}$

Summations $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$, $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$

Binomial theorem $(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r$ where $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

Maclaurin expansion $f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \ldots$

De Moivre's theorem $\left[r(\cos\theta + i\sin\theta)\right]^p = r^p(\cos p\theta + i\sin p\theta)$

Vector product $\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$

Matrix Transformation

Anti-clockwise rotation through an angle, θ about the origin, O $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Topic 1 Methods in Algebra and Calculus Assessment Standard 1.1	**Applying Algebraic Skills to partial fractions**			😊	😐	☹
I know that a **rational function** is a function which can be expressed in the form $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomial functions.						
I know that a **proper rational function** is a fraction where the **degree** of the **numerator** is **LESS** than the degree of the **denominator**.						
I can express a proper rational function as a sum of partial fractions whose denominator is of most degree 3 and easily factorised.						
Express each of the following in partial fractions by considering the type of denominator.						
Distinct Linear factors	♦ 1) $\frac{3x+2}{x+3 \quad x-4}$	♦ 2) $\frac{7x+1}{x^2+x-6}$	♦ 3) $\frac{8x+14}{(x-2)(x+1)(x+3)}$			
Repeated Factor	♦ 4) $\frac{3x+10}{(x+3)^2}$	♦ 5) $\frac{9}{(x-2)(x+1)^2}$	♦ 6) $\frac{x^2+6x-3}{x(x-1)^2}$			
Repeated Factor NOT factorised	♦ 7) $\frac{7x+33}{x^2-10x+25}$	♦ 8) $\frac{3x^2-5x-3}{x^2-x^3}$				
Linear factor and Irreducible Quadratic Factor	♦ 9) $\frac{x^2+2x+9}{(x-1)(x^2+3)}$	♦ 10) $\frac{7x^2-x+14}{(x-2)(x^2+4)}$	♦ 11) $\frac{5x^2-x+6}{x^3+3x}$			
I know that an **improper rational** function is a fraction where the degree of the numerator is **MORE** than or **EQUAL** to the degree of the denominator.						
I know how to **reduce an improper rational** function to a **polynomial** and a **proper rational function** using algebraic division.						
Express the following improper rational functions as a polynomial and a proper rational function which is given as partial fractions.						
➢ 12) $\frac{x^3+2x^2-2x+6}{(x-1)(x+3)}$	➢ 13) $\frac{x^2+3x}{x^2-4}$	➢ 14) $\frac{x^4+2x^2-2x+1}{x^3+x}$				

Topic 2 Methods in Algebra and Calculus Assessment Standard 1.2 **Applying calculus skills through techniques of differentiation**			
I can understand the method of differentiation from **first principles** using $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.			
I can differentiate an **exponential function** and I know that if $f(x) = e^x$ then $f'(x) = e^x$.			
I can differentiate a **logarithmic function** and I know that if $y = \ln x$ then $\frac{dy}{dx} = \frac{1}{x}$.			
I can differentiate a function using the **chain rule:** $(f(g(x))' = f'(g(x)).g'(x)$.			
I can differentiate a function using the **product rule:** $(f(x)g(x))' = f'(x)g(x) + f(x)\,g'(x)$.			
I can differentiate a function using the **quotient rule:** $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$.			
I can use the derivative of $\tan x$. If $f(x) = \tan x$ then $f'(x) = \sec^2 x$.			
I know that the **reciprocal trigonometric functions** are $\sec x = \frac{1}{\cos x}$, $\text{cosec}\, x = \frac{1}{\sin x}$ and $\cot x = \frac{1}{\tan x}$.			
I can derive and use the derivatives: $\frac{d}{dx}(\cot x) = -\cos ec^2 x$, $\frac{d}{dx}(\sec x) = \sec x \tan x$ and $\frac{d}{dx}(\cos\sec x) = -\cos\sec x \cot x$.			
Differentiate ♦ 1) $y = e^{3x}$ ♦ 2) $y = e^{4x^2 - 5x}$ ♦ 3) $y = \sqrt{e^{x^2} + 4}$ ♦ 4) $f(x) = \ln(x^3 + 2)$ ♦ 5) $f(x) = \sqrt{\sin 5x}$ ♦ 6) $f(x) = \sin^3(2x - 1)$ ♦ 7) $y = \frac{5x+2}{x-3}$ ♦ 8) $y = \frac{2x-5}{3x^2+2}$ ♦ 9) $y = 3x^4 sinx$ ♦ 10) $f(x) = x^2 \ln x, x > 0$ ♦ 11) $y = \frac{1+\ln x}{5x}$ ♦ 12) $y = \frac{\cos x}{e^x}$ ➢ 13) $y = e^{2x} \tan 3x$ ➢ 14) $f(x) = \ln\lvert\sin 2x\rvert$ ➢ 15) $y = \frac{\sec 2x}{e^{3x}}$ ➢ 16) $y = \frac{\tan 2x}{1+3x^2}$			

Topic 2 Methods in Algebra and Calculus Assessment Standard 1.2 **Applying calculus skills through techniques of differentiation**			
I know that $\frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}$.			
I know that $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$ are **inverse trigonometric functions**.			
I can differentiate an **inverse function** using $y=f^{-1}(x)\Rightarrow f(y)=x \Rightarrow (f^{-1}(x))'f'(y)=1 \Rightarrow (f^{-1}(x))'=\frac{1}{f'(y)}$.			
I know that $\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx}\cos^{-1}x=\frac{-1}{\sqrt{1-x^2}}$ and $\frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$			
I know using the **chain rule** that $\frac{d}{dx}\sin^{-1}(f(x))=\frac{f'(x)}{\sqrt{1-(f(x))^2}}$, $\frac{d}{dx}\cos^{-1}(f(x))=\frac{-f'(x)}{\sqrt{1-(f(x))^2}}$ and $\frac{d}{dx}\tan^{-1}(f(x))=\frac{f'(x)}{1+(f(x))^2}$			
Differentiate ♦ 17) $y=\sin^{-1}(3x)$ ♦ 18) $y=\sin^{-1}\left(\frac{x}{2}\right)$ ♦ 19) $y=\cos^{-1}(5x)$ ♦ 20) $y=\tan^{-1}\left(\frac{x}{4}\right)$ ➢ 21) $y=\tan^{-1}x^2$ ➢ 22) $y=2\sin^{-1}\sqrt{1+x}$ ➢ 23) $y=(x-3)\tan^{-1}(3x)$ ➢ 24) $y=\frac{\tan^{-1}2x}{1+4x^2}$			
I can find the **first** and **second derivative** of an **implicit function.**			
♦ 25) Find the first derivative of $x^3y+xy^3=4$ using implicit differentiation. ♦ 26) Find the equation of the tangent to the curve $y^3+2xy=x^2+4$, at the point (3, 1). ♦ 27) (a) Given $xy-x=4$, use implicit differentiation to obtain $\frac{dy}{dx}$ in terms of x and y. ➢ (b) Hence obtain $\frac{d^2y}{dx^2}$ in terms of x and y.			

Topic 2 Methods in Algebra and Calculus Assessment Standard 1.2 Applying calculus skills through techniques of differentiation			
I can find the **first and second derivative** of a **parametric function.**			
♦ 28) Given that $x=\ln(1+t^2), y=\ln(1+2t^2)$ use parametric differentiation to find $\frac{dy}{dx}$ in terms of t. ♦ 29) Given $x=\sqrt{t+1}$ and $y=\cot t$, $0<t<\pi$ obtain $\frac{dy}{dx}$ in terms of t. ♦ 30) (a) Given $y=t^3-\frac{5}{2}t^2$ and $x=\sqrt{t}$ for $t>0$ use parametric differentiation to express $\frac{dy}{dx}$ in terms of t in simplified form. ➢ (b) Show that $\frac{d^2y}{dx^2}=at^2+bt$, determining the values of the constants a and b.			
I can apply **parametric differentiation** to motion in a plane.			
♦ 31) At time t, the position of a moving point is given by $x=t+1$ and $y=t^2-1$. Find the speed when $t=2$. ♦ 32) The motion of a particle in the x-y plane is given by $x=t^2-5t, y=t^3-8t,$ where t is measured in seconds and x, y are measured in metres. Calculate the speed when $t=3$.			
I can use **logarithmic differentiation** when working indices involving the variable.			
I can use **logarithmic differentiation** when working with extended products and quotients.			
Use logarithmic differentiation to differentiate each of the following: ➢ 33) (a) $y=2^x$ (b) $y=x^{\tan x}$ (c) $y=\frac{x^2\sqrt{7x-3}}{1+x}$ ➢ 34) Given that $y=6^x\sqrt{1-2x}$, $x\geq\frac{1}{2}$, use logarithmic differentiation to find a formula for $\frac{dy}{dx}$ in terms of x.			

Topic 2 Methods in Algebra and Calculus Assessment Standard 1.2 Applying calculus skills through techniques of differentiation	😊	😐	☹
I can apply differentiation to related rates in problems where the functional relationship is given explicitly.			
➢ 35) The radius of a cylindrical column of liquid is decreasing at the rate of $0\cdot 02ms^{-1}$, while the height is increasing at the rate $0\cdot 01ms^{-1}$. Find the rate of change of the volume when the radius is 0·6 metres and the height is 2 metres. [Recall volume of a cylinder: $V=\pi\ r^2h$].			
➢ 36) Air is pumped into a spherical balloon at a rate of $48cm^3\ /\ s$. Find the rate at which the radius is increasing when the volume of the balloon is $\frac{32}{3}\pi cm^3$			
➢ 37) (a) A circular ripple spreads across a pond. If the radius increases at $0\cdot 1ms^{-1}$, at what rate is the area increasing when the radius is 8 cm? (b) If the area continues to increase at this rate, aw what rate will the radius be increasing when it is 5 metres?			

Topic 2 Applications of Algebra and Calculus Assessment Standard 1.5 Applying algebraic and calculus skills to problems	😊	😐	☹
I can apply differentiation to problems in context.			
➢ 1) A body moves along a straight line so that after t seconds its displacement from a fixed point O on the line is x metres. If $x=3t^2(3-t)$ find (a) the initial velocity and acceleration (b) the velocity and acceleration after 3 seconds. ➢ 2) A motorbike starts from rest and its displacement x metres after t seconds is given by: $x=\frac{1}{6}t^3+\frac{1}{4}t^2$. Calculate the initial acceleration and the acceleration at the end of the 2nd second.			
➢ 3) A cylindrical tank has a radius of r metres and a height of h metres. The sum of the radius and the height is 2 metres. (a) Prove that that the volume in m^3, is given by $V=\pi r^2(2-r)$. (b) Find the maximum volume.			

Topic 3 Applications of Algebra and Calculus Assessment Standard 1.1(a) — Applying algebraic skills to the binomial theorem			
I know and can use the notation $n!$ and ${}^nC_r$ where $n! = n(n-1)(n-2)(n-3)\ldots\ldots\ldots \times 3 \times 2 \times 1$ and ${}^nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$.			
I know Pascal's triangle up to $n=7$ and can apply the results $\binom{n}{r} = \binom{n}{n-r}$ and $\binom{n}{r-1} + \binom{n}{r} = \binom{n+1}{r}$.			
I know and can use the **Binomial Theorem** $(a+b)^n = \sum_{r=0}^{n} a^{n-r}b^r$ for r, $n \in N$ to expand an expression of the form $ax+b\;^{n}$ where $n \le 5, a, b \in Z$.			
I know that the **general term** in a binomial expansion $\binom{n}{r}x^{n-r}y^r$ can be used to find a **particular term** in a binomial expansion.			
I can expand an expression of the form $(ax^p + by^q)^n$, where $a, b \in Q; p, q \in Z; n \le 7$.			

- ♦ 1) Calculate $\dfrac{5!}{2}$
- ♦ 2) Calculate $\binom{8}{5}$
- ♦ 3) Simplify $\dfrac{(n+1)!}{(n-1)!}$
- ♦ 4) Write down $\binom{9}{3} + \binom{9}{4}$ as a binomial coefficient.
- ♦ 5) Solve, for $n \in N$, $\binom{n}{2} = 15$
- ♦ 6) Expand $(x+3)^4$
- ♦ 7) Expand $(2u-3v)^5$
- ♦ 8) Expand $(\frac{1}{2}x - 3)^4$ and simplify your answer.
- ♦ 9) (a) Write down the binomial expansion of $(1+x)^5$. (b) Hence show that $0\cdot9^5$ is 0·59049
- ➢ 10) Show that $\binom{n+1}{3} - \binom{n}{3} = \binom{n}{2}$ where the integer n is greater than or equal to 3.
- ➢ 11) Expand $\left(3x - \dfrac{1}{2x}\right)^6$
- ➢ 12) Find the coefficient of x^7 in $(\frac{2}{x} + x)^{11}$
- ➢ 13) Find the term independent of x in the expansion of $(3x^2 - \frac{2}{x})^9$
- ➢ 14) Write down the general term of the binomial expansion of $\left(2x^2y + \dfrac{4}{xy^2}\right)^6$. Use your expression to find the coefficient of $xy^{3}{}^{-3}$.

Topic 4 Applications of Algebra and Calculus Assessment Standard 1.1(b) **Applying algebraic skills to complex numbers**			
I know the definition of i as a solution of $x^2+1=0$, so $i=\sqrt{-1}$.			
I know the definition of the set of complex numbers as $C=\{a+bi : a,b\in R\}$ where a is the **real** part and bi is the **imaginary** part.			
I know that $z=a+bi$ is the **Cartesian form** of a complex number and that $\bar{z}=a-bi$ is the **conjugate** of z.			
I can perform addition, subtraction, multiplication and division operations on complex numbers.			
♦ 1) Solve $z^2=-9$ ♦ 2) Solve $z^2+2z+4=0$ ♦ 3) Solve $5z^2-4z+1=0$. ♦ 4) Calculate (a) $4-2i\ +\ 3+7i$ (b) $5+4i\ -\ 3-2i$ (c) $2-7i\ \ 3+2i$ (d) Divide $5+2i$ by $1-3i$ ♦ 5) Evaluate $\left(\frac{\sqrt{3}+i}{2}\right)^3$			
I know the **fundamental theorem of algebra** and the **conjugate roots** property.			
I can find the **roots of a quartic** when one complex root is given.			
I can **factorise polynomials** with real coefficients.			
I can find the **square root** of a complex number.			
I can **solve equations involving complex numbers** by equating real and imaginary parts.			
➢ 6) Show that $z=3+3i$ is a root of the equation $z^3-18z+108=0$ and obtain the remaining roots of the equation. ➢ 7) Given that $z=1-i$ is a root of the polynomial equation $z^4+4z^3-8z+20=0$, find the other roots. ➢ 8) Find the square roots of $5+12i$ ➢ 9) Calculate $\sqrt{8-6i}$ ➢ 10) Solve $z+i=2\bar{z}+1$ ➢ 11) Solve $z^2=2\bar{z}$ ➢ 12) Given the equation $z+2i\bar{z}=8+7i$, express z in the form $a+ib$.			

Topic 4 Geometry, Proof and Systems of Equations Assessment Standard 1.3 **Applying geometric skills to complex numbers**			
I can find the **modulus** and **principal argument** of a complex number given in **Cartesian** form.			
I know that $r(\cos\theta + i\sin\theta)$ is the **polar form** of a complex number.			
I can convert a given complex number from Cartesian to polar form or from polar to Cartesian form.			
♦ 1) Find the modulus and argument of : (a) $1+i\sqrt{3}$ (b) $1-\sqrt{2}i$ (c) $-5-5i$ ♦ 2) Write $z=-\sqrt{3}+i$ in polar form. ♦ 3) Write $z = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ in Cartesian form. ♦ 4) Given the equation $z=1-\sqrt{3}i$, write down $\bar{z}$ and express $\bar{z}^{-2}$ in polar form.			
I know and can use De Moivre's theorem with positive integer indices and fractional indices.			
I can apply De Moivre's theorem to multiple angle trigonometric formulae.			
I can apply De Moivre's theorem to find n^{th} roots of unity.			
♦ 5) Write the complex number $z=\sqrt{2}(1+i)$ in polar form and verify that z satisfies the equation $z^4+16=0$. ➢ 6) Let $z = \frac{(1+i)^9}{(1-\sqrt{3}i)^5}$. Find by using De Moivre's Theorem the modulus and argument of z. ➢ 7) Evaluate $z = \left(4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right)^{\frac{1}{2}}$ ➢ 8) Express $-i$ in the form $r(\cos\theta + i\sin\theta)$, where $-\pi < \theta \leq \pi$. Hence find the fourth roots of $-i$. ➢ 9) Solve $z^6=1$.			
I can plot complex numbers in the complex plane on an Argand Diagram.3			
I can interpret geometrically equations or inequalities in the complex plane of the form $\lvert z\rvert=1; \lvert z-a\rvert=b; \lvert z-i\rvert=\lvert z-2\rvert; \lvert z-a\rvert>b$.			
♦ 10) Show the complex numbers $z=3+4i$ and its conjugate, $\bar{z}$, on an Argand diagram. ➢ 11) Express $z=\frac{(1+2i)^2}{7-i}$ in the form $a+ib$, where a and b are real numbers. Show z on an Argand diagram and evaluate $\lvert z\rvert$ and $\arg(z)$. ➢ 12) Give a geometric interpretation and the Cartesian equation for each locus. (a) $\lvert z-2i\rvert=4$ (b) $\lvert z-1-3i\rvert\leq 5$ (c) $\lvert z-2\rvert=\lvert z+4i\rvert$			

Topic 5 Methods in Algebra and Calculus Assessment Standard 1.3 **Applying calculus skills through techniques of integration**

I can integrate expressions **using standard results**.

$\int f'(x)e^{f(x)}dx = e^{f(x)} + C$	$\int \frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert + C$	$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + C$	$\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Integrate the following:

♦ 1) $\int xe^{x^2}dx$ ♦ 2) $\int \frac{2x}{x^2+3}dx$ ♦ 3) $\int \frac{3x}{\sqrt{1-36x^4}}dx$ ♦ 4) $\int \frac{1}{25^2+x^2}dx$ ♦ 5) $\int \frac{3}{\sqrt{9-16x^2}}dx$

I can **Integrate by substitution** where the substitution is given.

♦ 6) Use the substitution $t = x^4$ to obtain $\int \frac{x^3}{1+x^8}dx$.

♦ 7) Integrate $\int \cos^3 x \sin x\, dx$ using the substitution $u = \cos x$.

♦ 9) Integrate $\int \sin^3 x \cos x\, dx$ using the substitution $u = \sin x$.

➢ 9) Use the substitution $u^2 = x - 2$ to obtain $\int \frac{x^2}{\sqrt{x-2}}dx$.

➢ 10) Find the value of $\int_4^9 \frac{2}{3+\sqrt{x}}dx$ using the substitution $u - 3 = \sqrt{x}$.

➢ 11) Integrate $\int \frac{6\sin x}{\sqrt{1-4\cos^2 x}}dx$ using the substitution $u = \cos x$.

➢ 12) Use the substitution $x = \sin u$ to obtain $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}dx$.

➢ 13) Use the substitution $x = 2\tan u$ to obtain $\int_2^{2\sqrt{3}} \frac{1}{4+x^2}dx$.

➢ 14) Use the substitution $x = 4\sin\theta$ to evaluate $\int_0^2 \sqrt{16-x^2}\,dx$.

♦ **Unit Assessment Standard** ➢ **Course Assessment Standard**

Topic 5 Methods in Algebra and Calculus Assessment Standard 1.3 — Applying calculus skills through techniques of integration	😊	😐	☹
I can use partial fractions to integrate **proper rational** functions where the denominator has **distinct linear** factors. ♦ 15) Integrate (a) $\int \frac{3}{(x-2)(x+1)}dx$ (b) $\int \frac{6x-11}{(x-3)(2x+1)}dx$ (c) $\int \frac{14-x}{x^2+2x-8}dx$ ➢ 16) Show that $\int_3^4 \frac{3x-5}{(x-1)(x-2)} = 2\ln 3 - \ln 2$ ➢ 17) Evaluate $\int_1^2 \frac{3x+5}{(x+1)(x+2)(x+3)}dx$ expressing your answer in the form $\ln\frac{a}{b}$, where a and b are integers.			
I can use partial fractions to integrate **proper rational** functions where the denominator has a **repeated linear** factor. ➢ 18) Integrate (a) $\int \frac{x}{(x-1)(x+1)^2}dx$ (b) $\int \frac{x^2-x-4}{(x+2)(x+1)^2}dx$ $\int \frac{x+1}{2x(x+3)^2}dx$ ➢ 19) Find the exact value of $\int_0^1 \frac{5x+7}{(x+1)^2(x+3)}dx$			
I can use partial fractions to integrate **improper rational** functions where the denominator has **distinct linear** factors. ➢ 20) Integrate (a) $\int \frac{x^2-6}{(x+4)(x-1)}dx$ (b) $\int \frac{3x^2-5}{x^2-1}dx$ 21) Find the exact value of $\int_0^2 \frac{2x^2-7x+7}{x^2-2x-3}dx$			
I can use partial fractions to integrate **proper** and **improper rational** functions where the denominator has a **linear factor** and an **irreducible quadratic** of the form x^2+a. ➢ 22) Find (a) $\int \frac{2x^2+1}{(x+1)(x^2+2)}dx$ (b) $\int \frac{2x^3-x-1}{(x-3)(x^2+1)}dx, x>3$ ➢ 23) Express $\frac{12x^2+20}{x(x^2+5)}$ in partial fractions. Hence evaluate $\int_1^2 \frac{12x^2+20}{x(x^2+5)}dx.$			

Topic 5 Methods in Algebra and Calculus Assessment Standard 1.3 Applying calculus skills through techniques of integration	😊	😐	☹
I can Integrate by parts using **one application**. ♦ 24) Use integration by parts to find: (a) $\int xe^x dx$ (b) $\int x\sin x dx$ ♦ 25) Evaluate (a) $\int_0^{\frac{\pi}{6}} x\cos x dx$ (b) $\int_0^1 xe^{2x} dx$			
I can Integrate by parts using a **repeated application**. ➢ 26) Use integration by parts to find: (a) $\int x^2 e^{3x} dx$ (b) $\int x^2 \cos 3x \, dx$ ➢ 27) Evaluate (a) $\int_1^2 x^2 \ln x dx$ (b) $\int_0^{\frac{\pi}{4}} e^{3x} \sin 2x dx$. ➢ 28) (a) Write down the derivative of $\sin^{-1} x$. (b) Use integration by parts to obtain $\int \sin^{-1} x . \frac{x}{\sqrt{1-x^2}} dx$. ➢ 29) Let $I_n = \int_0^1 x^n e^{-x} \, dx$ for $n \geq 1$. (a) Find the value of I_1. (b) Show that $I_n = nI_{n-1} - e^{-1}$ for $n \geq 2$. (c) Evaluate I_3.			

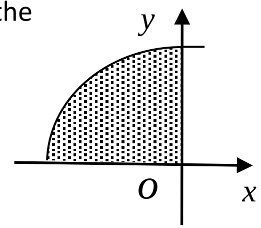
Topic 5 Applications of Algebra and Calculus Assessment Standard 1.5 Applying algebraic and calculus skills to problems	😊	😐	☹
I can apply integration to problems in context.			
➢ 1) The velocity, v, of a particle P at time t is given by $v = e^{3t} + 2e^t$. (a) Find the acceleration of P at time t. (b) Find the distance covered by P between $t = 0$ and $t = \ln 3$.			
➢ 2) An object accelerates from rest and proceeds in a straight line. At time, $t$seconds, its acceleration is given by $a = 20 - 2t$ cm/s^2. (a) Calculate the velocity of the object after 3 seconds. (b) How far did the object travel in the first 8 seconds of motion?			

Topic 5 Applications of Algebra and Calculus Assessment Standard 1.5 **Applying algebraic and calculus skills to problems**

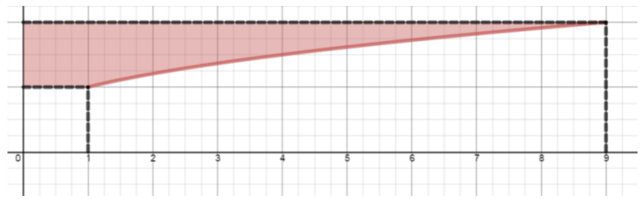
I can apply integration to the **evaluation of areas** including integration with respect to y.

➢ 3) The sketch on the right shows the graph of $y=\sqrt{3x+9}$.

Calculate the shaded area.



➢ 4) Calculate the area between the function $y=\frac{\sqrt{x}+1}{2}$ and the y-axis in the interval $1\le x\le 9$.



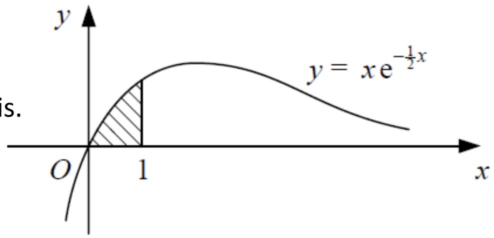
I can apply integration to **volumes of revolution**.

➢ 5) Sketch the curve $y=3x-x^2$ showing the roots. The region bounded by the curve and the x-axis is rotated through 360° about the x-axis.

Show that the volume of the solid generated $\frac{81}{10}\pi$.

➢6) The shaded region in the diagram, bounded by the curve $y=xe^{-\frac{1}{2}x}$ and the x-axis and the line $x=1$ is rotated through 360° about the x-axis.

Show that the volume of the solid generated $\pi(2-\frac{5}{e})$.



➢ 7) The area lying in the first quadrant and bounded by the curve $y=2x^2+1$, the y axis and the lines $y=2$ and $y=5$ is rotated 2π radians about the y axis, calculate the volume of the solid formed.

➢ 8) (a) Find the equation of the chord PQ which joins the points P(-2, 8) and Q(0, 16) on the curve $y=16-2x^2$.

(b) Show that the finite area enclosed between the curve $y=16-2x^2$ and the chord PQ is $\frac{8}{3}$ square units.

(c) Show that the volume generated when this area is rotated through 360° about the x-axis is $\frac{1024\pi}{15}$ cubic units.

Topic 6 Applications of Algebra and Calculus Assessment Standard 1.2 — **Applying algebraic skills to sequences and series**			
I know the formulae $u_n = a+(n-1)d$ and $S_n = \frac{1}{2}n[2a+(n-1)d]$ for the n^{th} term and the sum to n terms of an **arithmetic series**.			
I know the formula $u_n = ar^{n-1}$ and $S_n = \frac{a(1-r^n)}{1-r}, r \neq 1$ for the n^{th} term and the sum to n terms of a **geometric series**.			
I can apply the above rules on sequences and series to find:			
The n^{th} term The sum to n terms The common difference of arithmetic sequences The common ratio of geometric sequences			
I know and can use the formula $S_\infty = \frac{a}{1-r}$ for the **sum to infinity** of a geometric series where $\lvert r \rvert < 1$.			
I can expand $\frac{1}{1-r}$ as a **geometric series and extend to** $\frac{1}{a+b}$**.**			
➢ 1) The sum $S(n)$, of the first n terms of the sequence, $u_1, u_2, u_3 \ldots$, is given by $S(n) = 8n - n^2, n \geq 1$. Calculate the values of u_1, u_2, u_3 and state what type of sequence it is. Obtain a formula for u_n in terms of n, simplifying your answer.			
➢ 2) Given that $u_k = 11-2k,\ (k \geq 1)$, obtain a formula for $S_n = \sum_{k=1}^{n} u_k$. Find the values of n for which $S_n = 21$.			
➢ 3) A geometric series has the first term 2 and third term $\frac{1}{2}$. Find the value(s) of r, the common ratio, and any associated sum(s) to infinity.			
➢ 4) The arithmetic sequence a_1, a_2, a_3, a_4 .... and the geometric sequence b_1, b_2, b_3, b_4 .... both have their fifth term 8 and their eighth term 125. Find a_{15}. Calculate $\sum_{n=1}^{15} b_n$, correct to two decimal places.			
➢ 5) Given that there are two solutions find the third term of the geometric sequence whose second term is 400 and whose sum to infinity is 2500.			

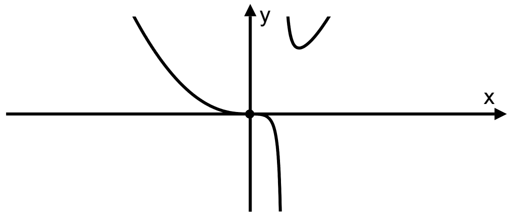
Topic 6 Applications of Algebra and Calculus Assessment Standard 1.2 **Applying algebraic skills to sequences and series**			
➢ 6) After an undetected leak at a nuclear power situation, a technician was exposed to radiation as follows: On the first day he received a dosage of 450 curie-hours On the second day he received a further dosage of 360 curie-hours On the third day he received a further dosage of 288 curie-hours (a) Show that these values could form the first 3 terns of a Geometric sequence and calculate how many curie-hours he was exposed to on the ninth day, assuming the pattern continues in the same way. (b) What was the **total radiation** received by him by day 15? (c) If the leak had continued undetected in this way, what would have been the final total long term exposure by the technician			
➢ 7) (a) The sum of the first 20 terms of an **arithmetic series** is 350. The first term is 4. (i) Calculate the common difference between terms. (ii) When did the sum first exceed 1000? (b) $x, x+6, x+k$ are the first 3 terms of a **geometric sequence**. (i) Write down an expression for the common ratio in two ways. (ii) Hence express x in terms of k. (iii) For what values of k will the sequence have a sum to infinity? (iv) Express the sum to infinity in terms of x. (v) For what **value** of x does this sum to infinity equal -24?			
➢ 8) Expand the following as geometric series and state the necessary condition on for each series to be valid. (a) $\frac{1}{1+x}$ (b) $\frac{1}{4-x}$ (c) $\frac{1}{3+x}$			
➢ 9) If S_n denotes the sum of the first n terms of the geometric series $1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\cdots$ where $x>1$ prove that $\frac{S_{2n}}{S_n}=1+\frac{1}{x^n}$.			
➢ 10) Find the common ratio of the geometric sequence $sin2\alpha, -sin2\alpha cos2\alpha, sin2\alpha cos^2 2\alpha, \ldots$. Prove that for $0<\alpha<\frac{\pi}{2}$ the series $sin2\alpha - sin2\alpha cos2\alpha + sin2\alpha cos^2 2\alpha + \cdots$ has a sum to infinity and that the sum to infinity is $tan\alpha$.			

Topic 6 Applications of Algebra and Calculus Assessment Standard 1.2 — Applying algebraic skills to sequences and series	🙂	😐	🙁
I know that a **power series** is an expression of the form: $f(x)=\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots\ldots a_n x^n + \ldots\ldots$ where $a_0, a_1, a_2 a_3, \ldots. a_n, \ldots..$ are constants and x is a variable.			
I understand and can use the **Maclaurin series**: $f(x)=\sum_{r=0}^{\infty} \frac{x^r}{r!} f^{(r)}(0)$ to find a power series for a simple non-standard function.			
I recognise and can determine the Maclaurin series expansions of the functions : e^x, $\sin x$, $\cos x$, $\ln(1 \pm x)$, knowing their range of validity $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\cdots$ $\quad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\cdots$ $\quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\cdots$ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots\cdots$ $\quad \ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots\cdots$			
♦ 1) Find the Maclaurin series expansions of the composite functions : (a) $\cos 3x$ (b) e^{x^3} (c) $e^{\sin x}$			
➢ 2) (a) Obtain the Maclaurin series for $f(x) = \sin^2 x$ up to the term in x^4. (b) Hence write down a series for $\cos^2 x$ up to the term in x^4.			
➢ 3) Find the Maclaurin expansion of $f(x) = \ln \ \cos x$, $0 \le x < \frac{\pi}{2}$, as far as the term in x^4.			
➢ 4) Write down the Maclaurin expansion of e^x as far as the term in x^4. Deduce the Maclaurin expansion of e^{x^2} as far as the term in x^4. Hence, or otherwise, find the Maclaurin expansion of e^{x+x^2} as far as the term in x^4 .			
➢ 5) Find the McLaurin expansion for $\frac{e^{2x}-1}{x}$ up to the term in x^4.			

Topic 7 Applications of Algebra and Calculus Assessment Standard 1.4 Applying algebraic and calculus skills to properties of functions			
I know the meaning of the terms function, domain / range, inverse function, stationary point, point of inflexion and local maxima and minima.			
I know the meaning of the terms global maxima and minima, critical point, continuous, discontinuous and asymptote.			
I can use the first derivative test for locating and identifying stationary points and horizontal points of inflexion.			
I can use the second derivative test for locating and identifying stationary points and non-horizontal points of inflexion.			
I can sketch the graphs of $\sin x, \cos x, \tan x, e^x \ln x$ and their inverse functions on a suitable domain.			
I know and can use the relationship between the graph of $y = f(x)$ and $y = kf(x)$, $y = f(x) + k$, $y = f(x+k)$, $y = f(kx)$ where k is a constant.			
I know and can use the relationship between the graph of $y = f(x)$ and $y = \lvert f(x) \rvert$, $y = f^{-1}(x)$.			
I can determine whether a function is **odd** or **even** or neither, and symmetrical and use these properties in graph sketching.			
I can sketch graphs of real rational functions using information, derived from calculus, zeros, asymptotes, critical points and symmetry.			
I know that the maximum and minimum values of a continuous function on a closed interval [a,b] can occur at stationary points, end points or points where f' is not defined.			
♦ 1) Sketch the graph of: (a) $y = \lvert \sin x \rvert \quad 0 \le x \le 2\pi$ (b) $y = \lvert 9 - x^2 \rvert \quad -6 < x < 6$.			
➢ 2) Determine whether $f(x) = x^2 \cos x$ is odd, even or neither.			
➢ 3) The function *f* is defined on the real numbers by $f \quad x = x^7 + \sin x$. Determine whether *f* is odd, even or neither.			
➢ 4) The function *f* is defined by $f \quad x = e^x \sin x$ where $0 \le x \le 2$. Find the coordinates of the stationary points of f and determine their nature.			
➢ 5) A function is defined by $g(x) = \frac{12\sqrt[3]{x}}{4x+1}, x \neq -\frac{1}{4}$. (a) Find the coordinates and nature of the stationary points of the curve with equation $y = g(x)$. (b) Hence state the coordinates of the stationary point pf the curve with equation $h(x) = \left\lvert \frac{12\sqrt[3]{x-1}}{4x-3} - 5 \right\rvert$.			

Topic 7 Applications of Algebra and Calculus Assessment Standard 1.4 **Applying algebraic and calculus skills to properties of functions**

➢ 6) The diagram shows part of the graph of $y=\dfrac{x^3}{x-2}, x\neq 2.$



(a) Write down the equation of the vertical asymptote.

(b) Find the coordinates of the stationary points of the graph of $y=\dfrac{x^3}{x-2}$.

(c) Write down the coordinates of the stationary points of the graph of $y=\left|\dfrac{x^3}{x-2}\right|+1$.

➢ 7) A function f is defined for suitable values of x by $f(x)=\dfrac{x^2-4}{1-x^2}$.

(a) Decide whether f is odd, even or neither.

(b) Write down the equations of any vertical asymptotes.

(c) Find algebraically the equation of any non-vertical asymptote.

(d) Show that f has only one stationary point and justify its nature.

(e) Sketch the graph of f, showing clearly what happens as $x\to\pm\infty$.

➢ 8) The function f is defined by $f(x)=3x+\dfrac{3}{x},\ x>0.$

(a) Write down an equation for each of the asymptotes of the graph of f.

(b) The graph of f has a stationary point when $x=a$. Find the coordinates of this stationary point and justify its nature.

(c) Sketch the graph of f.

(d) Find the volume of revolution formed when the region between $y=f(x)$, $x=a$, $x=3$ and $y=0$ is rotated 360° about the x-axis.

Topic 8 Applications of Algebra and Calculus Assessment Standard 1.3 Applying algebraic skills to summation and mathematical proof			
I know and can use the following sums of series: $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$, $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$.			
➢ 1) Find a **formula** for each of the following using the sum of series (a) $\sum_{r=1}^{n}(2r^2+3)$ (b) $\sum_{r=1}^{n}(r^2-6r)$ (c) $\sum_{r=1}^{n}(5r^2-2r-2)$ (d) $\sum_{r=1}^{n}(r^3+3r)$			
♦ 2) Evaluate each of the following using the sum of series: (a) $\sum_{r=1}^{20}(10r-1)$ (b) $\sum_{r=1}^{7}2r^2$ (c) $\sum_{r=1}^{100}(2k-3)^2$ (d) $\sum_{r=51}^{100}r^2$			
➢ 3) Express $\frac{2}{r^2+6r+8}$ in partial fractions. Hence evaluate $\sum_{r=1}^{n}\frac{2}{r^2+6r+8}$, expressing your answer as a single fraction.			
♦ 4) (a) Prove by induction that, for all natural numbers $n \geq 1$ $\sum_{r=1}^{n}3(r^2-r)=(n-1)n(n+1)$. (b) Hence evaluate $\sum_{r=11}^{40}3(r^2-r)$.			
♦ 5) Use Induction to prove that $\sum_{r=1}^{n} r^2+2r = \frac{1}{6}n \;\; n+1 \;\; 2n+7$ for all positive integers n.			
➢ 6) Use Induction to prove that $\sum_{r=1}^{n} 3^r = \frac{3}{2} \;\; 3^n-1$ for all positive integers n.			
➢ 7) Prove by induction that 2^{3n-1} is divisible by 7 for all integers $n \geq 1$.			
➢ 8) Prove by induction that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ for all integers $n \geq 1$.			
➢ 9) If $\mathbf{A}=\begin{pmatrix}2 & 1\\ -1 & 0\end{pmatrix}$, prove by induction that $\mathbf{A}^n=\begin{pmatrix}n+1 & n\\ -n & 1-n\end{pmatrix}$, where n is any positive integer.			

Topic 9 Geometry, Proof and Systems of Equations Assessment Standard 1.1(a) **Applying algebraic skills to systems of equations**			
I can use Gaussian elimination to solve a 3 × 3 system of linear equations and show that it has either: (a) a unique solution (b) has no solutions (inconsistency) or (c) has an infinite number of solutions (redundancy).			
I can compare the solutions of related systems of two equations in two unknowns and recognise ill-conditioning.			
♦ 1) Use Gaussian elimination to solve the system of equations: (a) $\begin{aligned} x+2y+z&=-3\\ 3x+y-z&=2\\ 2x+5y+3z&=-1 \end{aligned}$ (b) $\begin{aligned} 2x+y+z&=4\\ 2x-y-2z&=1\\ x-z&=2 \end{aligned}$ (c) $\begin{aligned} 3x-y-z&=-11\\ x-y+z&=-9\\ x+2y-2z&=9 \end{aligned}$			
➢ 2) (a) Apply the method of Gaussian elimination to $\begin{aligned} x+y+z&=2\\ x+2y+3z&=1\\ 3x+4y+5z&=5 \end{aligned}$ and show that there is an infinite number of solutions. (b) If a solution has $z=\lambda$, show that $y=-1-2\lambda$ and $x=3+\lambda$.			
➢ 3) For what values of a and b will the system of equations $\begin{aligned} x+2y+z&=60\\ 2x+3y+z&=85\\ 3x+y+pz&=105 \end{aligned}$ (a) be inconsistent and have no solutions (b) be redundant and have infinitely many solutions?			
➢ 4) A car manufacturer is planning future production patterns. Based on estimates of time, cost and labour, he obtains a set of three equations for the numbers, x, y and z of three new types of car. These equations are $\begin{aligned} x+2y+z&=60\\ 2x+3y+z&=85\\ 3x+y+(\lambda+2)z&=105, \end{aligned}$ where the **integer** λ is a parameter such that $0<\lambda<10$. (a) Use Gaussian elimination to find an expression for z in terms of λ. (b) Given that z must be a positive integer, what are the possible values for z ? (c) Find the corresponding values of x and y for each value of z .			
➢ 5) Determine if these systems of equations are ill conditioned. (a) $\begin{aligned} 7x+5y&=19\\ 4x+3y&=11 \end{aligned}$ and (b) $\begin{aligned} 7x+5y&=2\\ 4x-3y&=13 \end{aligned}$.			

Topic 9 Geometry, Proof and Systems of Equations Assessment Standard 1.1(b) **Applying algebraic skills to matrices**			
I know the meaning of the terms matrix, element, row, column, order, identity matrix, inverse, determinant, singular, non-singular and transpose.			
I can perform matrix addition, subtraction, multiplication by a scalar and multiplication.			
I know that: $A+B=B+A$ $AB\neq BA$ in general $(AB)C=A(BC)$ $A(B+C)=AB+AC$			
I know and can apply properties of the **transpose** matrix: $(A')'=A$ $(A+B)'=A'+B'$ $(AB)'=B'A'$			
I know and can apply properties of **the identity** and **inverse** matrix: $AA^{-1}=A^{-1}A=I$ $(AB)^{-1}=B^{-1}A^{-1}$			
I can calculate the determinant of 2 × 2 and 3 × 3 matrices. I can determine whether a matrix is **singular**.			
I know and can apply det (AB) = det A det B.			
I know and can find the inverse of a **non-singular** 2 × 2 matrix, $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ using $A^{-1}=\frac{1}{\lvert A\rvert}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.			
I can find the inverse, where it exists, of a 3 × 3 matrix using elementary row operations or the adjoint method.			
I can find the solution to a system of equations $AX=B$ where A is a 3 x 3 matrix and where the solution is unique.			
I know and can use the following 2 × 2 matrices to carry out single geometric transformations in the plane.			
Reflection in the *x*-axis $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Reflection in the line $y=x$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Reflection in the *y*-axis $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ Reflection in the line $y=-x$ $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ Rotation of θ radians in a positive direction about the origin $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ A dilatation about O, where the scale factor is k $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$			
I can identify and use the correct 2 × 2 transformation matrices, in the correct order, to carry out composite geometric transformations in the plane.			
♦ 1) Given the matrices $A=\begin{pmatrix} 4 & 8 \\ -2 & 6 \end{pmatrix}, B=\begin{pmatrix} -5 & 1 \\ 0 & 7 \end{pmatrix}, C=\begin{pmatrix} 3 & 0 \\ 10 & -4 \end{pmatrix}$ and $D=\begin{pmatrix} m & -3 & -8 \\ 0 & -2 & -1 \\ 1 & -1 & 4 \end{pmatrix}$ Find (*a*) $3A-2B+C$ (*b*) CB (*c*) A^{-1} (*d*) Determine the value(s) of m for which D is **singular.**			

Topic 9 Geometry, Proof and Systems of Equations Assessment Standard 1.1(b) Applying algebraic skills to matrices	☺	😐	☹
➢ 2) Calculate the inverse of the matrix $\begin{pmatrix} 2 & x \\ -1 & 3 \end{pmatrix}$. For what value of x is this matrix **singular**?			
➢ 3) Let A be the matrix $\begin{pmatrix} 3 & -2 \\ 5 & 9 \end{pmatrix}$. Show that $A^2 - 12A = nI$ where n is an integer and I is the 2×2 identity matrix.			
➢ 4) The matrix A is such that $A^2 = 4A - 3I$ where I is the corresponding identity matrix. Find integers p and q such that $A^4 = pA + qI$.			
➢ 5) (a) Given that $X = \begin{pmatrix} 2 & a \\ -1 & -1 \end{pmatrix}$ where a is a constant and $a \neq 2$, find X^{-1} in terms of a . (b) Given that $X + X^{-1} = I$, where I is the 2×2 the identity matrix, find the value of a .			
➢ 6) Matrices A and B are defined by $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} x+2 & x-2 & x+3 \\ -4 & 4 & 2 \\ 2 & -2 & 3 \end{pmatrix}$. (a) Find the product AB. (b) Obtain the determinants of A and of AB. Hence, or otherwise obtain an expression for det B.			
➢ 7) $A = \begin{pmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$ (a) Find A^{-1} the inverse of A. (b) X and B are two 2×2 matrices such that $AX = B$. Prove that $X = AB$			
➢ 8) A matrix is defined as $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & -1 \\ -4 & 1 & 1 \end{pmatrix}$. Show that matrix A has an inverse, A^{-1}, and find the inverse matrix.			
➢ 9) Write down the 2×2 matrix A representing a rotation of $\frac{\pi}{3}$ radians about the origin in an anticlockwise direction and the 2×2 matrix B representing a reflection in the y-axis. Hence, show that the image of the point (x, y) under the transformation A followed by the transformation B is $\left(-\frac{x-py}{2}, \frac{px+y}{2}\right)$, stating the value of p.			

Topic 10a Geometry, Proof and Systems of Equations Assessment Standard 1.4 **Applying algebraic skills to number theory**			
I can use Euclid's algorithm to find the **greatest common divisor** of two positive integers. ♦ 1) Use the Euclidean algorithm to obtain the greatest common divisor of 1139 and 629.			
I can express the greatest common divisor of the two positive integers as a linear combination of the two. ➢ 2) Use the Euclidean Algorithm to find integers x and y such that: (a) $210x + 156y = 6$ (b)$458x + 308y = 7$ (c) $3289x + 2415y = 23$			
I can use the division algorithm to write integers in terms of bases other than 10. ➢ 3) Use the division algorithm to express: (a) 468_{10} in base 7 (b) 999_{10} in base 6 (c) 1964_{10} in base 16			

Topic 10b Geometry, Proof and Systems of Equations Assessment Standard 1.5 **Applying algebraic and geometric skills to methods of proof**			
I understand and make use of the notations $\Rightarrow$, $\Leftarrow$ and $\Leftrightarrow$, know the corresponding terminology implies, implied by, equivalence.			
I know the terms natural number, prime number, rational number, irrational number.			
I know the terms if and only if, converse, negation and contrapositive.			
I can use direct proof. I can use indirect proof by providing a **counter-example**, using **proof by contradiction** or by using **proof by contrapositive**.			
♦ 1) Find a counterexample to disprove the conjecture that $x^2 - x > 0$ for all real values of x.			
♦ 2) Consider the statements **A** and **B**: **A** For any integer k, if $7k + 1$ is even, then k is odd ***B*** There is no largest even integer. (a) Prove statement *A* by considering its contrapositive. (b) Prove statement *B* by contradiction.			
♦ 3) Prove that the product of an odd and even integer is even.			
♦ 4) Prove by contradiction that if x is an irrational number, then $x - 7$ is irrational.			
♦ 5) Prove by contrapositive that if $\mathrm{x}^2 - 6\mathrm{x} + 5$ then x is odd.			
♦ 6) Given that $\mathrm{p}\ \mathrm{n}\ = \mathrm{n}^2 + \mathrm{n}$ consider the statements: **A** $p\ n$ is always even. **B** $p\ n$ is always a multiple of 3. For each statement, prove it is true, or otherwise, disprove it.			
♦ 7) Use the method of proof by contradiction to show that $\overline{x}$ is an irrational number.			

Topic 11 Geometry, Proof and Systems of Equations Assessment Standard 1.2 Applying algebraic and geometric skills to vectors	😊	😐	☹
I know the meaning of the term: Unit vector, Direction ratios, Direction cosines, Vector product, Scalar triple product.			
I can evaluate the vector product $\underline{a} \times \underline{b}$ using $\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \underline{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \underline{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \underline{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$ and I know that $(\underline{a} \times \underline{b}) = -(\underline{b} \times \underline{a})$.			
I know that the **magnitude** of the vector product $\lvert\underline{a} \times \underline{b}\rvert = \lvert\underline{a}\rvert\lvert\underline{b}\rvert\sin\theta$ which is the area of a parallelogram with sides $\underline{a}$, $\underline{b}$ and included angle θ .			
♦ 1) Given $\underline{u} = 5\underline{i} + \underline{j}$, $\underline{v} = 4\underline{i} - 2\underline{j} - 3\underline{k}$ and $\underline{w} = -2\underline{i} + 3\underline{j} + \underline{k}$ calculate : (a) $\underline{u} \times \underline{v}$ (b) $\underline{u} \times \underline{w}$ (c) $\underline{w} \times \underline{v}$ ➢ 2) Given $\underline{u} = 5\underline{i} + \underline{j}$, $\underline{v} = 4\underline{i} - 2\underline{j} - 3\underline{k}$ and $\underline{w} = -2\underline{i} + 3\underline{j} + \underline{k}$ calculate : (a) $\underline{u}.(\underline{v} \times \underline{w})$ (b) $\underline{w}.(\underline{v} \times \underline{u})$			
➢ 3) Three vectors OA, OB and OC are given by $\underline{u}, \underline{v}$ and $\underline{w}$ where $\underline{u} = \underline{i} + 5\underline{j} + \underline{k}$, $\underline{v} = 2\underline{i} + 3\underline{k}$ and $\underline{w} = -\underline{i} + 5\underline{j} - 2\underline{k}$. Calculate $\underline{u}.(\underline{v} \times \underline{w})$. Interpret your result geometrically.			
I can find the equation of a line in parametric, symmetric or vector form.			
I can find the angle between two lines in three dimensions.			
I can determine whether or not two lines intersect and, where possible, find the point of intersection.			
♦ 4) Find, in vector, parametric and symmetric form an equation for the line which passes through the points (3, –2, 4) and (2, 5, –2).			
➢ 5) Find the acute angle between the lines $\underline{r} = \begin{pmatrix} 7 \\ 1 \\ 5 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ and $\underline{r} = \begin{pmatrix} -2 \\ 6 \\ -3 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$.			
➢ 6) Let L_1 and L_2 be the lines $L_1 : x = 8 - 2t, y = -4 + 2t, z = 3 + t$ and $L_2 : \frac{x}{-2} = \frac{y+2}{-1} = \frac{z-9}{2}$. (a) Show that L_1 and L_2 intersect and find their point of intersection. (b) Verify that the acute angle between them is $\cos^{-1}\left(\frac{4}{9}\right)$.			

Topic 11 Geometry, Proof and Systems of Equations Assessment Standard 1.2 **Applying algebraic and geometric skills to vectors**			
I can find the equation of a plane in vector form, parametric form or Cartesian form.			
I can find the point of intersection of a plane with a line which is not parallel to the plane.			
I can determine the intersection of 2 or 3 planes.			
I can find the angles between a line and a plane or between 2 planes.			
♦ 7) Find, in Cartesian form, the equation of the plane which has normal vector $\begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix}$ and passes through the point (4, –7, 2).			
➢ 8) Find an equation of the plane π which contains the points A (1, 1, 0), B(3, 1, -1) and C(2,0,-3).			
➢ 9) Find the point of intersection of the line $\frac{x-3}{4}=\frac{y-2}{-1}=\frac{z+1}{2}$ and the plane with equation $2x+y-z=4$.			
➢ 10) Find an equation for the line of intersection of the plane with equation $2x+y+4z=6$ and the plane with normal vector $\underline{i}+\underline{j}-2\underline{k}$ through the point (1,1,1).			
➢ 11) Find the angle between the line $x=12t+1$, $y=3t-4$, $z=-t+5$ and the plane $2x-3y+4z=3$.			
➢ 12) Find the angle between the line $\frac{x-3}{2}=\frac{4-y}{5}=\frac{2z+3}{2}$ and the plane $x+2y-3z=2$.			
➢ 13) Find the angle between the two planes with equations $2x-y+z=5$ and $x+y-z=1$, respectively.			
➢ 14) (a) Find the equation of the plane containing the points A(2, 0, 1), B(3, 1, 0) and C(0, 1, 1). (b) Find the angle between this plane and the plane with equation $2x+y+z=1$. (c) Find the point of intersection of the plane containing A, B, and C and the line with equation $\frac{x-1}{2}=\frac{y+4}{2}=\frac{z-1}{1}=t$.			

Topic 12 Methods in Algebra and Calculus Assessment Standard 1.4 — Applying calculus skills to solving first order differential equations	😊	😐	🙁
I can solve first order differential equations of the form $\frac{dy}{dx}=g(x)h(y)$ or $\frac{dy}{dx}=\frac{g(x)}{h(y)}$ by **separating the variables**. I can find **general** and **particular solutions** given suitable information.			
♦ 1) Solve $\frac{dy}{dx}=y(x-1)$ ♦ 2) Solve $\frac{dy}{dx}=2x(1+y^2)$ ♦ 3) Solve $\frac{dy}{dx}=4xe^{-y}$ ➢ 4) For a differential equation $\frac{dx}{dt}=(3-x)(1+x)$, when $x=0, t=0$, show that $\frac{1+x}{3-x}=Ae^{kt}$, stating the values of A and k. Hence express the solution explicitly in the form $x=f(t)$. ➢ 5) Solve $\frac{1}{x}\frac{dy}{dx}=y\sin x$ given that when $x=\frac{\pi}{2}$, $y=1$. ➢ 6) Solve the differential equation $\frac{dy}{dx}=\frac{\sqrt{x}}{e^{3y}}$, given $y=0$ when $x=1$ expressing y explicitly in terms of x. ➢ 7) Solve the differential equation giving y in terms of x: $\cos y\frac{dy}{dx}=x^2\operatorname{cosec}x^2 y$, given that when $x=\frac{1}{2}$, $y=\frac{\pi}{2}$.			
i can solve first order linear differential equations given or rearranged in the form $\frac{dy}{dx}+p(x)y=f(x)$ using the **integrating factor method**. I can find **general** and **particular solutions** given suitable information.			
♦ 8) Solve $\frac{dy}{dx}+\frac{3y}{x}=\frac{e^x}{x^3}$ ♦ 9) Solve $\frac{dy}{dx}+y\cot x=\cos x$ ♦ 10) Solve $x^2\frac{dy}{dx}+3xy=\sin x$ ➢ 11) Solve $x\frac{dy}{dx}-y=x^2$ given that $y=3$ when $x=1$. ➢ 12) Solve $\frac{dy}{dx}=y\tan x-\sec x$ given that when $x=0$, $y=1$.			

Topic 12 Methods in Algebra and Calculus Assessment Standard 1.4 Applying calculus skills to solving second order differential equations			
I know the meaning of the terms homogeneous, non-homogeneous, auxiliary equation, complementary function and particular integral.			
I can find the **general solution** of a second order **homogeneous** ordinary differential equation $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$ with constant coefficients where the roots of the auxiliary equation are (a) real and distinct (b) real and equal (c) are complex conjugates. ♦ 1) Solve the equations: (a) $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$ (b) $\frac{d^2y}{dx^2}+12\frac{dy}{dx}+9y=0$ (c) $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+13y=0$.			
I can solve **initial value problems** for second order **homogeneous** ordinary differential equation with constant coefficients. ➢ 2) Solve $\frac{d^2y}{dx^2}+12\frac{dy}{dx}+36y=0$ with $y=0$ and $\frac{dy}{dx}=1$ when $x=1$. ➢ 3) Solve $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+13y=0$ with $y=-1$ and $\frac{dy}{dx}=2$ when $x=0$.			
I can solve second order **non-homogeneous** ordinary differential equation with constant coefficients $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f(x)$ using the auxiliary equation and particular integral method.			
➢ 4) Obtain the **general solution** of the differential equation $\frac{d^2y}{dx^2}-7\frac{dy}{dx}+10y=23\sin x+11\cos x$. ➢ 5) (a) Find the **general solution** to the following differential equation: $\frac{d^2y}{dx^2}-4\frac{dy}{dx}-5y=1-x^2$. (b) Hence find **the particular solution** for which $y=0$ and $\frac{dy}{dx}=-18$ when $x=0$. ➢ 6) Solve the second order differential equation $3\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=x^2$, given that when $x=0$, $y=2$ and $\frac{dy}{dx}=3$. ➢ 7) Solve the second order differential equation $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=3e^{2x}$, given that when $x=0$, $y=-1$ and $\frac{dy}{dx}=-1$.			